A Critique of Two Models for Cyclone Performance

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This note reviews two models for the particle collection efficiency of gas cyclones. The model developed by Leith and Licht (1972) has been advocated widely (e.g., Koch and Licht, 1977; Cooper, 1983; Licht, 1984, 1988). As shown here, however, Leith and Licht's development contains two fundamental difficulties concerning the particle collection rate and mean gas residence time. The model equations are corrected in this note. The other model considered is that of Dietz (1981), whose derivation is also shown to contain a difficulty corrected in this note. The grade efficiency curves predicted from the corrected form of Dietz' model are shown to be realistic. These two models are also compared with the approach developed by Mothes and Löffler (1984).

To facilitate comparison, equations developed by earlier authors are assigned the same numbers as in the original publications, with prefixes *L*, *D*, and *M* for equations given by Leith and Licht, Dietz, and Mothes and Löffler, respectively. Figure 1, which shows a typical reverse flow cyclone, defines the symbols used to denote the cyclone dimensions. Both Leith and Licht, and Dietz distinguish between the "entry region" (annulus between the cyclone barrel and the exit pipe or "vortex finder") and the "collection region," which comprises the cyclone barrel and part of the cone of the cyclone up to the "natural length" of the vortex as defined by Alexander (1949).

Particle Motion: Leith and Licht Model

Leith and Licht describe particle motion in the entry and collection regions using an equation of particle motion based on several simplifying assumptions which include the following:

• Gas path lines are circles with velocity profile intermediate between free and forced vortex flow.

• The angular velocities of gas and particles are equal everywhere, thus making the particle/gas slip velocity purely radial.

• The radial slip gives rise to a radial drag force, acting toward the axis, with instantaneous drag on a particle equal to the steady drag at the instantaneous slip velocity (i.e., a "quasisteady drag" assumption) and given by the Stokes law.

• The radial acceleration of a particle is negligible, thus making the drag force equal to the mass of the particle times its centripetal acceleration.

The radial migration velocity of a particle is then:

$$\frac{dR}{dt}=\frac{d_p^2\,\rho_p}{18\mu}\,\frac{u_{T2}^2\,R_2^{2n}}{R^{2n+1}}=fn.\ (R) \tag{L3}$$

where the particle of diameter d_p and density ρ_p in question is at radius R after spending time t within the cyclone. [This form is a corrected version of a formula by Leith and Licht (1972), which incorrectly equates the power of R_2 as $(2n+2)$. This, however, appears to have been a printing error, since it does

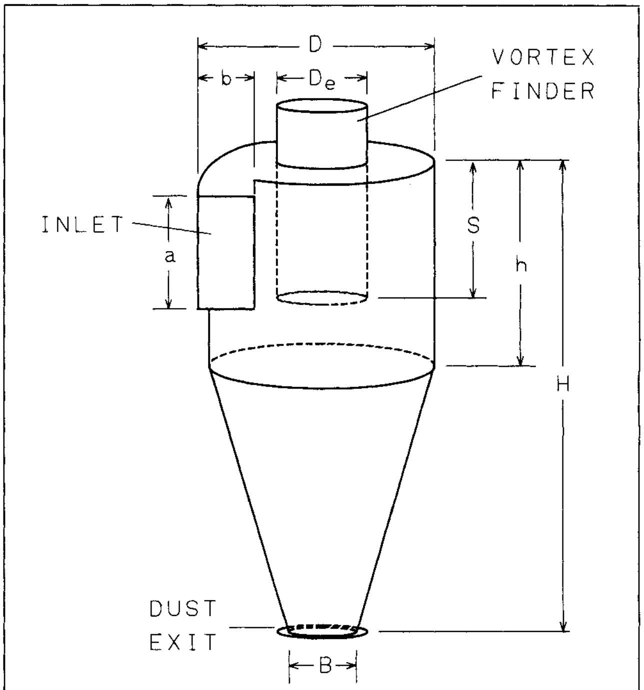

Figure 1. Cyclone with tangential inlet.

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not apply to relationships derived from Eq. L3.] The radius of the cyclone wall is R_2, the gas viscosity is μ, and u_{T2} is the tangential gas inlet velocity. The physical significance of Eq. L3 is that, because of the simplifying assumptions made, the radial migration velocity of a particle depends on its position in the cyclone, but not on its starting position or the length of time to reach the current position.

Leith and Licht integrate Eq. L3 to obtain the particle trajectories in the cyclone, broadly following earlier analyses (e.g., Davies, 1952; Strauss, 1975). They also estimate the particle collection rate as the particle flux to the wall, contending that a particle which contacts the wall is removed from the gas. In the analysis of Leith and Licht, this flux is calculated in terms of the residence time of particles in the cyclone, assuming that the particles follow the deterministic trajectories obtained from Eq. L3. Leith and Licht assume further that axial mixing within the cyclone is negligible, but that radial mixing of particles is complete: the particle concentration c is a function of z, the axial position in the cyclone, but independent of R. Our first objection to their analysis is that the assumption of complete radial mixing means that the particle position is random, not a deterministic function of initial position and time spent in the cyclone. In other words, the assumptions made by Leith and Licht are inconsistent. In fact, this inconsistency is unnecessary, since the collection rate can be calculated from the basic assumptions simply by evaluating the migration velocity at the wall from Eq. L3 as dR/dt at $R=R_2$. The resulting expression developed below, however, differs from that of Leith and Licht. Before the analysis is pursued further, another difficulty in the Leith and Licht model must be examined—the mean gas residence time. Furthermore, there is a difficulty in Leith and Licht's manipulation of their equations (see Appendix), so that relaxing the inconsistency in the assumptions would not make their model valid.

Gas Residence Time

Leith and Licht (1972) argue that, because the gas enters the entry region of the cyclone distributed over the height of the inlet duct, on average it enters a distance $a/2$ below the top of the cyclone. Dietz (1981) effectively makes the same assumption, in reducing the length of the entry region by $a/2$ because the gas is distributed. In each case, the mean gas residence time in the entry region is estimated as:

$$t_1=\frac{V_s'}{Q} \tag{1}$$

where Q is the volumetric gas throughput and V_s' is the volume of the entry region below the centerline of the inlet duct:

$$V_s'=\pi(S-a/2)\,(D^2-D_e^2)/4 \tag{2}$$

Equation 2 is incompatible with Danckwerts' (1953) result that, regardless of the configuration of inlet or outlet, the mean residence time of fluid in a steady, continuous flow system is given by the system volume divided by the volumetric throughput. It may readily be shown that Danckwerts' general result applies to a system with distributed feed. Therefore, Eq. 2 should be replaced by:

$$V_s=\pi S(D^2-D_e^2)/4 \tag{3}$$

which is not only correct, but simpler. Noting that normally $a \simeq S$, it follows that both Leith and Licht (1972) and Dietz (1981) underestimate t_1 by about a factor of two.

Leith and Licht extend this argument to the collection region, where the gas flow from the outer vortex into the central vortex is distributed along the "natural length" of the core. They assume that the flow per unit length from the outer to the inner vortices is constant, and hence infer that the mean residence time in the outer vortex is equal to half of its volume divided by the volumetric flow rate. Danckwerts' general result, however, also applies to a system with distributed outflow. Therefore, Leith and Licht underestimate the mean residence time in the collection region t_2 by exactly a factor of two.

The total mean residence time in the cyclone, $t_{res} = t_1+t_2$, is correctly calculated as the cyclone volume, from inlet to vortex finder, divided by the gas throughput. The Leith and Licht estimate for t_{res} is approximately half of the correct value.

Inserting the correct expression for the residence time in the entry region in the formulae of Dietz has relatively little effect on the calculated grade efficiency curves. A fundamental reworking of the derivations of Dietz using a distributed inlet is possible and was carried out, but the predicted grade efficiency curves for this case also fell very close to those of the original model.

Correcting the gas residence time in the Leith and Licht model to the larger value found above, but accepting the equations of Leith and Licht, results in the prediction of a higher separation efficiency for a given cyclone. A specific example of this effect is shown in Figure 2. However, it is more constructive to rework the Leith and Licht model with the corrected expression for the particle removal rate.

Revised Leith and Licht Model for Particle Removal

From Eq. L3, the particle migration velocity at the wall of the cyclone, $R=R_2$, is:

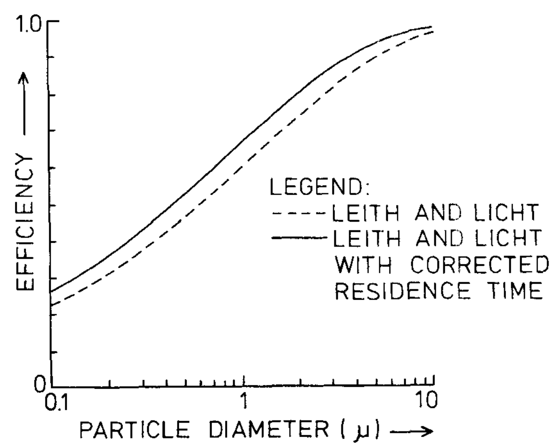

Figure 2. Effect of correcting the gas residence only on the calculated-grade efficiency curves in the Leith and Licht model.

Stairmand high-efficiency cyclone: diameter, 0.2 m; powder density, 2,640 kg/m³

$$v_2 \equiv \left.\frac{dR}{dt}\right|_{R=R_2} = \frac{d_p^2\,\rho_p\,u_{T2}^2}{18\mu\;\;R_2} \tag{4}$$

which is the result obtained and applied by Dietz (1981). Broadly using the Leith and Licht method, but with a less circuitous derivation, we now consider a section across the cyclone at which the number of particles per unit volume of gas is c. The number of particles contacting the unit area of the cyclone wall per unit time is then cv_2. Thus, over a length dz of the cylindrical part of the cyclone, the rate of particle removal is $2\pi R_2 c v_2 dz$. These particles are collected from a volume $\pi R_2^2 dz$ of gas within this element of the cyclone so that

$$\frac{d}{dt}\left[\pi R^2 c dz\right] = -2\pi R_2 c v_2 dz \tag{5}$$

Simplifying, substituting for v_2 from Eq. 4, and rearranging, Eq. 5 becomes:

$$\frac{d(\ln c)}{dt} = -\frac{d_p^2\,\rho_p\,u_{T2}^2}{9\,\mu\,R_2^2} \tag{6}$$

Calling the inlet particle concentration c_o and integrating Eq. 6 from $t=0$ to $t=t_{\text{res}}$, the mean residence time in the cyclone yields an expression for the particle concentration in the exit gas stream c. The efficiency for particles of diameter d_p and density ρ_p becomes:

$$\eta = \frac{c_o - c}{c_o} = 1 - \exp\left[-\frac{\rho_p}{9\mu}\left(\frac{d_p\,u_{T2}}{R_2}\right)^2 t_{\text{res}}\right] \tag{7}$$

which may be compared with the corresponding result from Leith and Licht:

$$\eta = 1 - \exp\left[-2\left\{\frac{\rho_p}{9\mu}\left(\frac{d_p\,u_{T2}}{R_2}\right)^2 (n+1)\,t'_{\text{res}}\right\}^{1/(2n+2)}\right] \tag{L19a}$$

where, in addition to the differences in the algebraic form, the Leith and Licht estimate for mean gas residence time t'_{res} is approximately half of the correct value t_{res} for the reasons outlined above. Thus, the correct expression, Eq. 7, is again simpler than Leith and Licht's result, Eq. L19a, but does not contain the "vortex index," n, i.e., the distribution of tangential velocities in the cyclone.

Particle Migration between Annular and Central Vortices

The Dietz model includes an explicit expression for exchange of particles across the boundary between the annular vortex and the central vortex core given by:

$$\Gamma_v = c_2(z)\;u_{R,e} - c_4(z)\;v_{R,e} \tag{D3}$$

where $c_2(z)$ is the particle concentration in the annulus, $c_4(z)$ is that in the inner vortex core, $u_{R,e}$ is the radial slip velocity of gas entering the core region, and $v_{R,e}$ is the radial slip velocity (due to centrifugal action) of particles immediately inside the core. Dietz evaluated $v_{R,e}$ by applying Eq. L3 at the radius of the vortex finder and neglecting the effect of $u_{R,e}$.

The net particle flux between the annular and core regions in Eq. D3 is thus evaluated as a balance between two fluxes: 1. outwards neglecting the radial inflow of gas and 2. inwards neglecting the centrifugal particle motion. Dietz adopted this approach to overcome the difficulty in defining the flux if a "conventional" approach following Barth (1956) was used to obtain the particle velocity.

However, there are two fundamental problems in Eq. D3. First, it implies that the particle concentration changes discontinuously across the boundary between the two regions, which is physically unrealistic. Furthermore, the whole approach of considering the transport of particles to be the net result of two opposing fluxes does not correctly reflect the physical transport process. To better describe the process of particle exchange between the two regions, which are still at different concentrations, Mothes and Löffler (1984) introduced a particle diffusivity between the regions. The net particle flux is then obtained as:

$$\Gamma_v = D_p\,\frac{c_2(z) - c_4(z)}{R_a^* - R_e} + [u_{R,e} - v_{R,e}]c_j\,(z) \tag{M14}$$

where R_a^* and R_e are the radii of the annular and core regions, respectively. $c_j(z)$ is equal to $c_4(z)$ or $c_2(z)$, depending on whether $v_{R,e}$ is larger or smaller than $u_{R,e}$. The first term of Eq. M14 effectively represents radial particle dispersion with D_p the effective particle dispersion coefficient, while the second term represents convective migration. Equation M14 is conceptually an improvement on Eq. D3, although it introduces a new parameter D_p, which in general is unknown.

Predictions vs. Experiments

To demonstrate the effects of these corrections on the models, their predictions are compared in Figures 3 and 4 with experimental grade efficiency data reported in the literature. These figures are represented in the form suggested by Abrahamson and Allen (1986) who showed that the experimental data for collection of solid particles in cyclones of different geometries

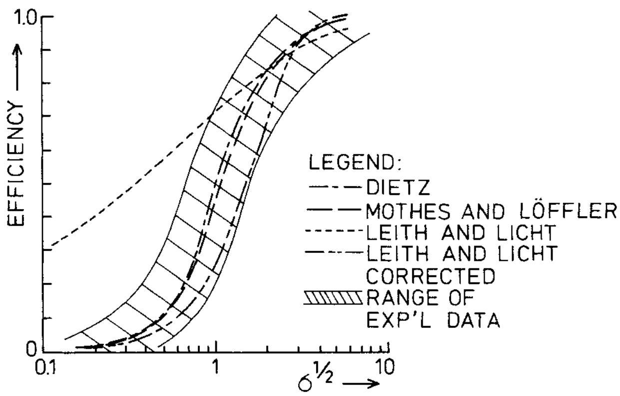

Figure 3. Model predictions of the performance of a Stairmand high-efficiency cyclone vs. band of experimental results for various geometries according to Abrahamson and Allen (1986).

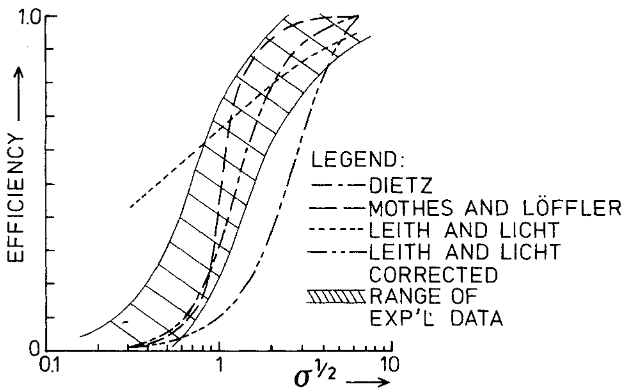

Figure 4. Model predictions of the performance of a modern commerical design of cyclone vs. band of experimental results for various geometries according to Abrahamson and Allen (1986).

can be accommodated in a single plot by showing the collection efficiency as a function of the ratio of (outward) radial slip velocity to (inward) radial gas velocity at the radius of the vortex finder. In terms of the parameters in Eqs. D3 and M14, this ratio is:

$$\sigma = v_{R,e}/u_{R,e} \tag{8}$$

For the cases examined by Abrahamson and Allen, $v_{R,e}$ was estimated by a modification of the method of Barth (1956). They modified the method of Barth in two ways: 1. they used the gas flow field calculated according to the more sophisticated method of Meissner (1978), instead of following Barth and 2. they allowed for particle Reynolds numbers outside the creeping flow range in calculating $v_{R,e}$ from the gas flow field. The results reported by Abrahamson and Allen, which were compiled for a wide range of cyclones of industrial scale and design operated at ambient conditions, fall within the hatched bands in Figures 3 and 4. Abrahamson and Allen also showed that the effects of increasing temperature and pressure are adequately accounted for by representing results with σ as the normalizing parameter.

The reliability of the models can now be assessed by determining whether their predictions for common cyclone designs are consistent with the band of data. Their predictions for a Stairmand and a van Tongeren cyclone, both of "high-efficiency" design, are shown in Figures 3 and 4, respectively.

The original version of the Leith and Licht model completely fails to reflect the shape of experimental grade efficiency curves. As shown in Figure 2, correcting the residence time alone does not remove the discrepancy. The fully corrected version of their model, represented by Eq. 7 above, predicts the dependence on particle size better, but even with the larger estimate for gas residence time the collection efficiency is still underpredicted. Note that the corrected version of this model insufficiently accounts for the effect of the "vortex finder" diameter D_e which only appears in the Leith and Licht model through its weak effect on gas residence time. In reality, D_e is one of the most important design variables in cyclones, and reducing D_e increases η (Perry et al., 1984). The cyclone design of Figure 4 uses a low value for D_e, and the efficiency is seen to be strongly underpredicted in this case.

The models of both Mothes and Löffler and of Dietz reflect the general experimental trends very well. However, in view of the lack of physical relevance of Eq. 10, the satisfactory predictions from the model of Dietz should probably be regarded as fortuitous rather than as a necessary consequence of the formulation of the model. An estimate of D_p, the particle dispersion coefficient and its variation with geometry, scale and operating conditions must be made for the model of Mothes and Löffler to be of practical use.

Notation

a,b = height and width of tangential gas inlet
A = cross-sectional area of flow channel
B = diameter of dust exit
c = number concentration of particles in gas
c_o = number concentration of particles in gas entering cyclone
$c_2(z)$ = number concentration of particles in annulus
$c_4(z)$ = number concentration of particles in vortex core
D = diameter of cyclone barrel
D_e = diameter of "vortex finder"
D_p = particle dispersion coefficient
d_p = particle diameter
d_{p50} = diameter of particle separated with an efficiency of 50%
H = overall height of cyclone
h = height of cyclone barrel
n = index describing velocity distribution in gas vortex
Q = volumetric gas throughput
R = radial position of particle in cyclone
R_1 = radial position of particle entering cyclone
R_2 = radius of cyclone wall
S = length of vortex finder
t = time elapsed since entry of particle into cyclone
t_1 = mean gas residence time in entry region
t_2 = mean gas residence time in collection region
t_{res} = mean gas residence time $(t_1 + t_2)$
u_R = radial gas velocity
$u_{R,e}$ = radial velocity of gas entering vortex core
u_{T2} = tangential inlet velocity
$v_{R,e}$ = radial slip velocity of particle just inside vortex core
v_2 = particle migration velocity at wall of cyclone
V_s = volume of entry region of cyclone
V_s' = volume of entry region below centerline of inlet duct
z = axial position in cyclone

Greek letters

Γ_v = particle flux from annulus to vortex core
η = collection efficiency of cyclone
μ = viscosity of gas
ρ_p = density of particle
σ = ratio of velocities defined in Eq. 8

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Appendix: Particle Migration Velocity

From Eq. L3, Leith and Licht derive an expression for the radial migration velocity for particles starting with a hypothetical initial position on the axis of the cyclone:

$$\frac{dR}{dt}=\frac{\rho_p}{18\mu}\left(\frac{d_p u_{T2}}{R_2}\right)^2 R_2\left[\frac{\rho_p(n+1)}{9\mu}\left(\frac{d_p u_{T2}}{R_2}\right)^2 t\right]^{-\frac{(2n+1)}{(2n+2)}} \qquad \text{(L18)}$$

The reason for using this hypothetical initial position is not stated. Equation L18 gives dR/dt for any value of the residence time of the hypothetical particle in the cyclone, and therefore applies to any R. However, to obtain the radial migration velocity at the wall, it must be applied at the specific value of t, t_{res}, corresponding to $R=R_2$. Leith and Licht proceed to obtain the collection rate by integrating (dR/dt) at $R=R_2$ with respect to time, correctly interpreted as the residence time of gas in the cyclone. Their residence time equation is:

$$\int_{n_0}^{n'}\frac{dn'}{n'}= -\int_0^t \frac{\rho_p}{9\mu}\left(\frac{d_p u_{T2}}{R_2}\right)^2\left[\frac{\rho_p(n+1)}{9\mu}\left(\frac{d_p u_{T2}}{R_2}\right)^2 t\right]^{-\frac{(2n+1)}{(2n+2)}} dt \qquad \text{(L19)}$$

where n' is the number of particles remaining in the gas. It readily may be shown that Eq. L19 leads only to Leith and Licht's equation for collection efficiency (Eq. L19) if t in the integrand is treated as a variable. Moreover, as noted above, the appropriate value of dR/dt is obtained only for the specific particle residence time $t=t_{\text{res}}$, because only at this time the particle is at $R=R_2$.

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